Sequences 2

Friday, April 7, 2023 8:57 AM

(ch. 10.1 thm 1, 2, 3, 4, 5, 6)

thm 1: $\lim_{n\to\infty} (a_n \ (+,-,\times,\div)\ b_n) = \lim_{n\to\infty} a_n \ (+,-,\times,\div) \lim_{n\to\infty} b_n$ ($\lim_{n\to\infty} b_n \neq 0$) $\qquad \lim_{n\to\infty}(ka_n) = k \lim_{n\to\infty} a_n$

thm 2: (sandwich) if $a_n \to L$ & $c_n \to L$ then ANY b_n such that $\boxed{a_n \le b_n \le c_n}$ has $b_n \to L$

thm 3: (limits & continuity) if $a_n \to L$ & f continuous (@ L) then $f(a_n) \to f(L)$

} convergence

* write thm in exam *

thm 4: f(x) defined for $x \ge n_0$, $a_n = f(n)$ where $n \ge n_0$ then

$\lim_{n\to\infty} a_n = L$ if $\lim_{x\to\infty} f(x) = L$

$\lim_{n\to\infty} f(n) = L$

* when a_n following f(n) function *

thm 5: common limits (look @ book)

ex 1) (thm 2) $b_n = \dfrac{\ln(n)\cdot 2^n}{n!}$ (product could be > n!) (n!: product of integers 1 → n)

$0 \le b \to$ choose $a_n = 0$ (constant sequence = 0)

$$\frac{\ln(n)\cdot 2^n}{n!} = \ln(n)\cdot \underbrace{\overbrace{\frac{2}{1}}^{2}\cdot\overbrace{\frac{2}{2}}^{1}\cdot\overbrace{\boxed{\overset{(n)}{\cdots}\ \frac{2}{n-1}}}^{\le 1}\cdot\frac{2}{n}}_{\text{numerators} = 2\times2\times2\ldots,\ \text{denominators} = n!} \le \underbrace{\frac{\ln(n)}{n}\cdot 4}_{(c_n)} \to 0$$

$\le \left(\ln(n)\cdot 2\cdot 1\cdot \frac{2}{n}\right)$

$a_n \le b_n \le c_n$, since a_n & $c_n \to 0$ means $b_n \to 0$

ex 2) $\cos\left(\ln\left(\underbrace{\overbrace{1+\frac{1}{n}}^{x}}_{a_n \to 1 \text{ as } n\to\infty}\right)\right) \longrightarrow 1 = f(L)$ with $L = 1$ & $f(x) = \cos(\ln(x))$

reason for divergence (unboundedness) (should):

- $\lim_{n\to\infty} a_n = \infty$ or $-\infty$
- a_n oscillating

bound doesn't necessarily mean limit / bound just contains sequence / ceiling & floor

thm: if a_n converges then a_n is bounded

* bounded above: # > than all (ceiling)
bounded below: # < than all (floor)
bounded : both below & above *

* if bounded, not always converging *

thm: a_n unbounded then a_n diverges

* if divergent, not always unbounded *

ex 1) $a_n = 3^n$ since for any $m \in \mathbb{R}^+$ (+ real #'s) exists $n \in \mathbb{N}$ (natural #) such that $3^n \ge m \to (a_n)$ not bounded (above)

* can continuously ↑ *

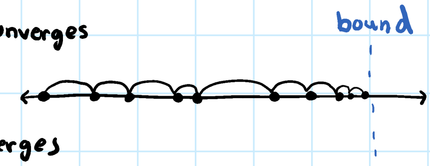
Monotone Convergence (thm 6): may show convergence (doesn't compute L)

thm 6 (MCT):
- if a_n bounded above (↑ above) & + increasing → a_n converges

bound

thm 6 (MCT):

- if a_n bounded above & + increasing $\rightarrow$ a_n converges
  (↑ above)



- if a_n bounded below & − decreasing $\rightarrow$ a_n converges
  (↓ below)